

Non-supersymmetric meta-stable vacua from brane configurations

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ABSTRACT: We construct configurations of NS-, D4-, and D6-branes in type IIA string theory, realizing the recently discussed non-supersymmetric meta-stable minimum of 4d $\mathcal{N} = 1$ $SU(N_c)$ super-Yang-Mills theories with massive flavors. We discuss their lift to M-theory and the mechanism of pseudo-moduli stabilization. We extend the construction to many other examples of meta-stable minima, including the SO/Sp theories, $SU(N_c)$ with matter in two-index tensor representations, and to a chiral gauge theory.

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1. Introduction

The realization in [1] that simple theories like 4d $\mathcal{N} = 1$ SYM theories with massive flavors contain non-supersymmetric local meta-stable minima has triggered a lot of interest in this phenomenon, and in particular in its realization in string theory.¹ One possible embedding is by considering configurations of D-branes at a Calabi-Yau singularity [3] (for earlier related work on supersymmetry breaking in this kind of configurations, see [4–6]), or

¹Meta-stable non-supersymmetric vacua had been discussed in the supersymmetry model building literature, see e.g. [2].

wrapped on suitable cycles at a local Calabi-Yau [7].² This has led to interesting steps in the construction of string theory models of gauge mediated supersymmetry breaking [10] (along the lines in [11]).

In this paper we explore a different realization, in terms of type IIA configurations of NS-branes with D4-branes suspended between them, in the presence of D6-branes, and of their lifts to M-theory. The realization of $\mathcal{N} = 1$ SYM theories with flavors in this setup, and the realization of Seiberg duality, are standard [12] (see [13] for a review), so they provide a solid starting point for the discussion. We use these tools to construct the non-supersymmetric local meta-stable minimum of SYM theories with unitary, orthogonal or symplectic gauge groups (with massive flavors).

The lift to M-theory of the configurations corresponding to the supersymmetric vacua of these theories has also been extensively discussed (see e.g. [14–16] for $SU(N_c)$ SYM). In these cases, the configuration lifts to a single smooth M5-brane wrapped on a holomorphic curve, which encodes important information concerning the non-perturbative infrared dynamics of the theory.

A natural question is whether the information about the low energy dynamics of the local meta-stable minima is also encoded in the M5-brane curve. We describe the main properties of the M-theory lift of the type IIA configuration realizing this minimum. The lift corresponds to a reducible M5-brane geometry, with two components which are holomorphic in different complex structures of the underlying geometry (which is a Taub-NUT hyper-Kähler geometry), and hence are volume-minimizing by themselves. One of the components has a number of free parameters, which correspond to pseudo-moduli of the effective field theory. The mechanism that lifts the pseudo-moduli is not encoded in the geometry of the curve, but rather these flat directions are removed only when the interaction between the two components (described in the large distance regime in terms of exchange of gravitons and 3-form fields) is taken into account. Hence, the 1-loop stabilization in the field theory maps to a process beyond the M5-brane probe approximation in the M-theory configuration. This problem is very difficult with present techniques, hence we simply sketch the main points, hoping for further progress in the future.

The realization of known non-supersymmetric local meta-stable minima in terms of brane configuration leads to a precise identification of the key ingredients in this phenomenon. This allows for many generalizations, and we present explicit construction illustrating just a few. We provide the type IIA brane configurations corresponding to new non-supersymmetric local meta-stable minima in $SU(N_c)$ theory with non-chiral matter in symmetric or antisymmetric representations (plus massive flavors), and in a chiral $SU(N_c)$ theory with chiral multiplets in the antisymmetric, conjugate symmetric and fundamental representations (plus massive flavors).

The paper is organized as follows. In section 2 we review the field theory description of the non-supersymmetric meta-stable vacua of [1]. In section 3 we describe the type IIA configuration realizing the non-supersymmetric vacuum in the $SU(N_c)$ theory with massive

²Gauge sectors of the kind described in [1] have been embedded also in heterotic compactifications in [8, 9]. However, the existence of local meta-stable minima in these constructions, where gravity is not decoupled, remains an open question.

flavors, and discuss its classical properties. In section 4 we describe the M-theory lift of this configuration and discuss the physics encoded (and not encoded) in the M5-brane curve. In section 5 we describe the physical mechanism lifting the classical pseudo-moduli of the non-supersymmetric vacuum, and its realization in string/M -theory. In section 6 we introduce O4-planes in the type IIA configurations to realize the non-supersymmetric vacua in the SO and Sp theories. Further generalizations are constructed in section 7. Section 8 contains our final remarks.

As we were finishing this paper [18] appeared, which overlaps with the results of sections 3 and 5. After we finished this paper [19] appeared, which clarifies some aspects of the discussion in section 4.

2. The ISS model

In this section we sketch the analysis in [1] to determine the existence of meta-stable vacua in $\mathcal{N} = 1$ $SU(N_c)$ SYM with massive flavors. We refer the reader to this reference for details.

Consider $SU(N_c)$ SYM with N_f massive flavors Q, \tilde{Q} with mass much smaller than Λ , the dynamical scale of the gauge theory. Since the analysis is carried out in the dual theory, we work on the free magnetic range $N_c + 1 \leq N_f < \frac{3}{2}N_c$ so that the latter is IR free and the Kähler potential is under control in the small field region.

The dual theory is $SU(N)$ SYM with $N = N_f - N_c$, with N_f flavors q, \tilde{q} and the mesons Φ . They transform as $(\square, \bar{\square}, 1), (\bar{\square}, 1, \square), (1, \square, \bar{\square})$ under the $SU(N) \times SU(N_f) \times SU(N_f)$ color and flavor symmetry.

The superpotential is of the form

$$W = h \text{Tr} (q \Phi \tilde{q}) - h\mu^2 \text{Tr} \Phi \tag{2.1}$$

(where the traces run over flavor indices).

This theory breaks supersymmetry at tree level due to the F-term of Φ (the so-called rank condition). There is classical moduli space of minima, parametrized by the vevs

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix}, \quad q = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \quad \tilde{q}^T = \begin{pmatrix} \tilde{\varphi}_0 \\ 0 \end{pmatrix}, \quad \text{with } \tilde{\varphi}_0 \varphi_0 = \mu^2 \mathbf{1}_N. \tag{2.2}$$

The computation of the Coleman-Weinberg one-loop effective potential shows that all pseudo-moduli (classical flat directions not corresponding to Goldstone directions) are lifted in the one-loop effective potential, and that the maximally symmetric point in the classical moduli space

$$\Phi_0 = 0, \quad \varphi_0 = \tilde{\varphi}_0 = \mu \mathbf{1}_N, \tag{2.3}$$

is a minimum of the one-loop effective potential.

As mentioned in [1], in the case of different flavor masses the local minimum is obtained by setting the N non-zero dual quark vevs equal to the N largest masses. If a dual quark vev is set to be one of the N_c smallest masses, the configuration is unstable already at the

classical level, due to the appearance of a negative mass squared mode (which triggers the rolling to the correct minimum).

The $SU(N)$ gauge dynamics is IR free and hence not relevant in the small field region, but it is crucial in the large field region. In fact, it leads to the appearance of the $N_f - N$ supersymmetric vacua predicted by the Witten index in the electric theory. In the large field region of Φ vevs, $|\mu| \ll |\langle h\Phi \rangle|$, the N_f flavors are very massive, and we recover pure $SU(N)$ SYM dynamics, with a dynamical scale Λ' given by

$$\Lambda'^{3N} = \frac{h^{N_f} \det \Phi}{\Lambda^{N_f - 3N}} \tag{2.4}$$

where Λ is the Landau pole scale of the IR free theory. The complete superpotential, including the non-perturbative $SU(N)$ contribution is

$$W = N (h^{N_f} \Lambda^{-(N_f - 3N)} \det \Phi)^{1/N} - h\mu^2 \text{Tr} \Phi \tag{2.5}$$

This superpotential leads to $N_f - N$ supersymmetric minima at

$$\langle h\Phi \rangle = \Lambda_m \epsilon^{2N/(N_f - N)} \mathbf{1}_{N_f} = \mu \frac{1}{\epsilon^{(N_f - 3N)/(N_f - N)}} \mathbf{1}_{N_f}, \tag{2.6}$$

where $\epsilon \equiv \frac{\mu}{\Lambda}$. In the regime $\epsilon \ll 1$, the vevs are much smaller than the Landau pole scale, and the analysis can be trusted. Notice also that these minima sit at $|\langle h\Phi \rangle| \gg |\mu|$, hence very far from the local non-supersymmetric minimum. This separation and the height of the potential barrier make the meta-stable SUSY breaking vacuum long-lived.

3. Type IIA configuration

An efficient way to realize supersymmetric gauge field theories in string theory is to embed them as the effective gauge theory on the world-volume of configurations of D- and NS-branes. These Hanany-Witten setups [20] have been successfully employed in the study of four-dimensional gauge theories with $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supersymmetry (see e.g. [12, 23, 14–16] and [13] for a review with more complete references). In this section we describe the type IIA brane configuration that corresponds to the ISS non-supersymmetric local minimum, and understand some of its classical properties.

A convenient starting point is the type IIA brane configuration of $\mathcal{N} = 1$ $SU(N_c)$ SYM with N_f flavors.³ We consider one NS-brane stretching along the coordinates 012345, one NS-brane (denoted NS'-brane) stretching along 012389, N_c color D4-branes stretching along 0123 and suspended in x^6 between the NS- and NS'-branes, and N_f flavor D6-branes stretching along 0123789. See [12, 13] for more details. We consider the configuration for zero flavor masses (namely, the D6's have all the same position as the NS' in 45). The configuration is shown in figure 1a.

³This configuration can be obtained from the one describing $\mathcal{N} = 2$ SQCD with $SU(N_c)$ gauge group by rotating the NS' (originally parallel to the NS) [21]. The relative angle θ between the NS and NS' branes dictates the mass of the adjoint chiral superfield $|\mu(\theta)| = \tan \theta$. In the limit in which the NS and NS' become orthogonal $\mu \rightarrow \infty$. Integrating out the adjoint chiral superfield we are left with $\mathcal{N} = 1$ $SU(N_c)$ SQCD with vanishing superpotential. Although this viewpoint is useful in the derivation of the M-theory lift of the type IIA configuration, in this section we directly discuss the final rotated configuration.

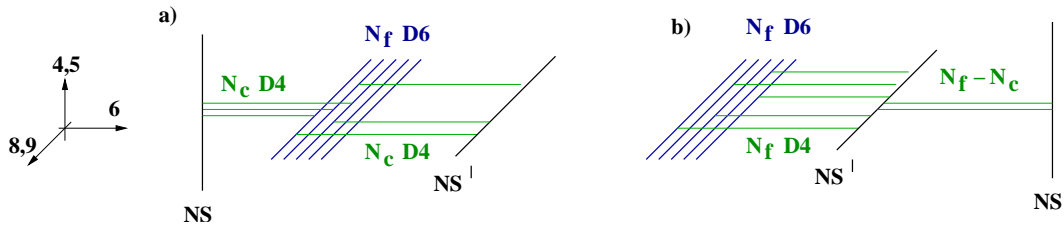


Figure 1: The type IIA brane configurations for $SU(N_c)$ SYM with N_f (massless) flavors (a) and its Seiberg dual theory (b).

Notice that the D4-branes can in general split in pieces as they are able to end on the D6-branes. Notice also the familiar s-rule [20] which forbids that there are at most one D4-brane piece connecting the NS-brane with a given D6-brane. The number of D4-brane pieces connecting the NS'-brane with a given D6-brane is on the other hand arbitrary.

Following the operations in [12, 13], it is straightforward to obtain the brane configuration describing the Seiberg dual theory. Sketchily, one considers moving the NS across the D6-branes (process in which the N_c finite D4-brane pieces joining them disappear, and $N_f - N_c$ new finite D4-brane pieces appear), and then across the NS'. The final configuration is shown in figure 1b. Notice the familiar realization of the meson vevs as the position in 8,9 of the N_f D4-branes pieces suspended between the D6-branes and the NS'-brane.

3.1 The SUSY breaking minimum

Let us now consider the type IIA configurations and the above processes in the presence of non-zero flavor masses, by moving off the D6-branes in the directions 45. Consider for simplicity the case where all flavor masses are equal.

The introduction of flavor masses corresponds in the magnetic field theory to the introduction of the linear term in the mesons that triggers supersymmetry breaking. Recall that there is a non-supersymmetric set of vacua, where the dual quarks have non-trivial vevs (fixed by the flavor masses), and which is parametrized by pseudo-moduli encoded in an $N_c \times N_c$ block of the mesonic matrix.

These features are nicely reproduced by the type IIA configuration. When the D6-branes are moved off in 45, the $N_f - N_c$ D4-branes joining them to the NS-brane move along 45 and maintain the same supersymmetry. However, the N_f D4-brane pieces joining them to the NS'-brane misalign with respect to them, leading to a non-trivial F-term. The F-term can be partially canceled by recombining $N_f - N_c$ of such D4-branes with the D4-branes joining the D6- and the NS-branes. This recombination corresponds to the fact that $N = N_f - N_c$ entries in the dual quarks acquire non-zero vevs to minimize the F-term. Notice that the appropriate breaking of the global symmetry is nicely reproduced. For shortness, we sometimes denote D4'-branes the D4-branes suspended between the D6- and the NS'-brane.

The configuration for the supersymmetry-breaking configuration is shown in figure 2.⁴

⁴This configuration is the starting point of our studies. It has been well investigated in the past in order to study the deformation corresponding to adding terms linear in the mesons to the magnetic superpotential

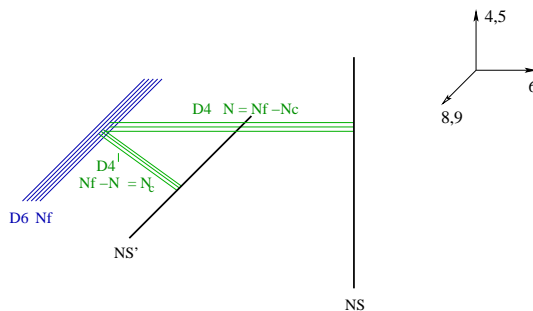


Figure 2: Type IIA brane configuration corresponding to the SUSY breaking minimum.

Notice that the $N_f - N_c$ D4'-branes joining the D6-branes to the NS'-brane are free to move in the directions 8, 9, hence reproducing (most of) the classical moduli space of non-supersymmetric vacua of the field theory (notice that the pseudo-modulus θ is not manifest in the geometry⁵).

It is very interesting to consider more general situations, with arbitrary non-zero masses. Recall that in the field theory analysis in [1], the non-supersymmetric vacua are obtained when the vevs for the dual quarks $\varphi_0, \tilde{\varphi}_0$ are given by the N largest masses (out of the N_f mass parameters). In configurations where some vev is given by one of the N_c smallest masses, a classically unstable mode appears.

This behavior is easily reproduced by the type IIA configuration. The different flavor masses correspond to different 4, 5 positions for the different D6-branes. The brane setup corresponding to the classical non-supersymmetric configuration suggested in ISS is shown in figure 3. In this configuration, the N_c D4-branes connected to the NS'-brane end on the N_c D6-branes which are closest (i.e. those associated with the N_c smallest mass parameters). This is in order to minimize the energy of the configuration. In addition, the remaining N D4-branes connected to the NS-brane, end on the farthest N D6-branes (i.e. those associated with the N largest mass parameters). The reason for this is clarified in the next paragraph. Recall that the position in 4, 5 of these D4-branes is related to the dual quark vevs, so we have indeed found the configuration realizing the ISS vacuum for different masses.

It is now easy to realize what goes wrong if one considers the configuration where a dual quark vev is given by one of the N_c smallest masses. Clearly, there is one D6-brane on which two D4-brane pieces (one connecting to the NS- and other to the NS'-branes) coincide. Since these D4-branes are non-supersymmetric with respect to each other, an open string tachyon develops at their intersection. This is precisely the unstable mode which appears in the field theory analysis. Notice that the stretched open string leading to the tachyon is a component of the meson field Φ , in agreement with this interpretation.⁶

(see for example [13]. The only new ingredient is to allow the rank of the quark mass matrix of the electric theory (linear couplings in the magnetic dual) to be larger than N_c . This brane setup was discussed by various attendants to a group meeting at the Institute for Advanced Study.

⁵The pseudo-modulus θ is defined such that $\langle \varphi_0 \rangle = \mu e^\theta \mathbf{1}_N$ and $\langle \tilde{\varphi}_0 \rangle = \mu e^{-\theta} \mathbf{1}_N$ [1].

⁶One may be surprised by the fact that an open string tachyon is captured by a field theory analysis.

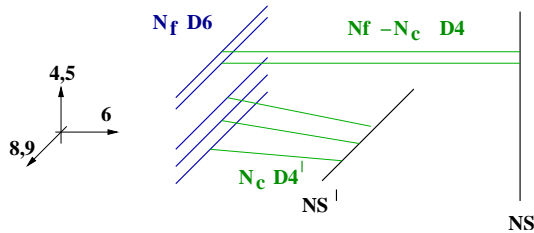


Figure 3: The non-supersymmetric type IIA configuration reproducing the non-supersymmetric ISS field theory minimum for arbitrary flavor masses.

Notice also that this tachyon only appears at the origin in the mesonic (pseudo)moduli space, in agreement with the field theory analysis. However one cannot try to avoid this classical instability by moving off in the (pseudo)moduli space, since quantum corrections (see later) lift it dynamically pushing the configuration towards the origin.

The above brane configuration can also be used to study the situation of SQCD with $N_{f,0}$ massless and $N_{f,1}$ massive flavors. These models have been discussed in [3] from the field theory viewpoint. In particular, it was shown that the magnetic dual exhibits supersymmetry breaking by the rank condition for $N_{f,0} < N_c$, and that in this case one does not have a meta-stable minimum, but rather a saddle point with a runaway direction parametrized by the mesons formed by the massless quarks (and which becomes the runaway triggered by the Affleck-Dine-Seiberg superpotential in the large field region).

The above brane configuration provides a simple explanation of these facts. We consider the type IIA brane configuration in which $N_{f,0}$ D6-branes sit at the origin in the directions 4, 5. If $N_{f,0} < N_c$, then $N = N_f - N_c < N_{f,1}$ and there are some of the $N_{f,1}$ D6-branes associated with non-zero masses which are not endpoints of the N D4-branes in the D4/NS5 system. These D6-branes can be used as endpoints of the D4-branes in the D4/NS' system and lead to non-supersymmetric configurations. If on the other hand $N_{f,0} > N_c$, then $N > N_{f,1}$ and the N D4-branes in the D4/NS system occupy all the $N_{f,1}$ massive flavor D6-branes (and some more). Hence the D4-branes in the D4/NS' system are forced to end on the massless flavor D6-branes, leading to a final supersymmetric configuration. Thus one reproduces the above mentioned condition to have rank supersymmetry breaking.

The above discussion leads to an important observation. There is a dynamical ‘s-rule’ in the non-supersymmetric configurations of our interest, which prevents a D4-brane and a D4'-brane to end on the same D6-brane. Although more manifest in the case of different masses, this conclusion is general and valid in the case of equal masses. This has an important implication on the structure of the M5-brane describing the M-theory lift of our type IIA configurations.

3.2 The SUSY breaking minimum in the electric theory

Once we have identified the structure of the supersymmetry breaking meta-stable mini-

In fact, similar phenomena occur in other non-supersymmetric tachyonic D-brane configurations, in the regime of small supersymmetry breaking, see e.g. [22].

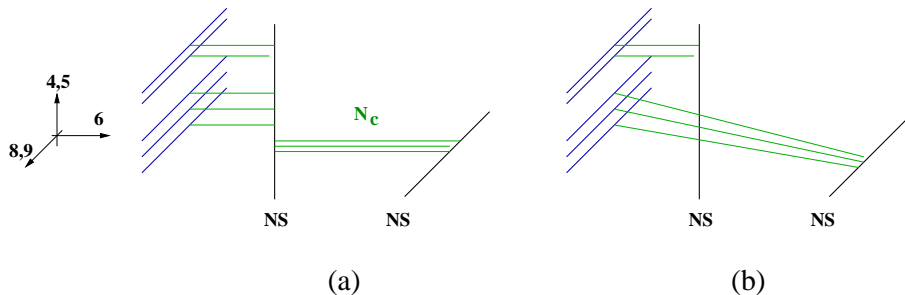


Figure 4: Figure (a) shows the brane configuration describing the supersymmetric minimum of the electric theory. Figure (b) shows the supersymmetry breaking meta-stable vacua in the brane configuration realizing the electric theory.

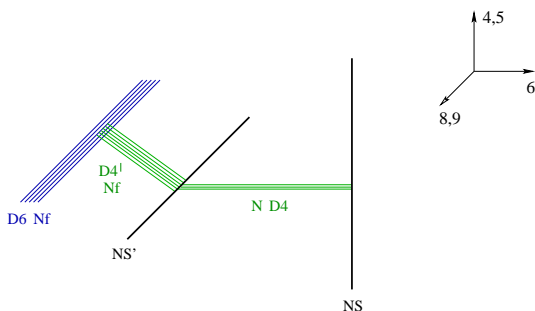


Figure 5: Brane configuration describing the $\varphi_0 = \tilde{\varphi}_0 = 0$ point used to estimate the height of the potential barrier.

imum in the brane configuration realizing the magnetic theory, it is possible to obtain it in the brane realization of the electric theory. This is simply obtained by undoing the Seiberg duality, namely by crossing back the NS'- and NS-branes. The resulting configuration is shown in figure 4.b. The supersymmetric configuration corresponding to the supersymmetric minima of the theory is shown in figure 4.a.

3.3 Longevity of the meta-stable SUSY breaking vacuum

As discussed in [1], the longevity of the meta-stable SUSY breaking vacuum depends on its distance to the SUSY vacua in field space and the height of the potential barrier separating them. Both of them can be estimated by considering a simple trajectory connecting the minima.

The separation between vacua is determined by the expectation value of Φ at the supersymmetric minimum, the type IIA brane setup provides a simple visualization of the barrier height. The $\varphi_0 = \tilde{\varphi}_0 = 0$ point corresponds to no recombination of the D4-branes. The increase in length of the branes accounts for the additional potential energy.

4. M-theory lift

In the realization of 4d gauge theories using type IIA brane configurations, many interesting properties of the field theory are unveiled by lifting the configurations to M-theory [23, 14 –

16] (see [13] for a review). The supersymmetric vacua of the theory are easily determined in M-theory. Theories that exhibit dynamical supersymmetry breaking have been studied in this context in [17]. In this section we describe the M-theory lift of our type IIA configurations. For simplicity we focus on the situation where all flavor masses are equal (generalization to different masses is straightforward when the N largest masses are arbitrary, but the N_c D6-branes corresponding to the smallest masses are coincident, see later).

4.1 The factorization

In the lift to M-theory, D4-branes and the NS- or NS'-branes on which they end become different parts of a single smooth M5-brane, wrapped on a complex curve in the ambient space (which is given by an N_f -centered Taub-NUT geometry, corresponding to the M-theory lift of the N_f D6-branes). In this section we investigate what is the M-theory configuration that describes the SUSY breaking meta-stable minimum. From the structure of the type IIA configuration corresponding to the non-supersymmetric vacuum, one can draw an important conclusion: the M5-brane curve in the lift of the configuration is split into two components. This follows from the fact that the D4-branes ending on the NS-brane are completely disconnected from the D4-branes ending on the NS'-brane. This implies that the D4/NS system and the D4'/NS' lift to two independent M5-brane curves.

Notice moreover that in the case of equal masses each system by itself preserves some supersymmetry (more precisely, in order for this to happen the D6-branes connected to D4'-branes must be coincident). Namely, the D4/NS system preserves 8 supercharges in the presence of the D6-branes, while the D4'/NS' system preserves 4 supercharges in the presence of the D6-branes.⁷ Notice however that the supersymmetries preserved by both systems are not compatible, and the complete configuration breaks all supersymmetries.

This structure has a beautiful counterpart in the M-theory lift. The geometry in the M-theory lift is given by a Taub-NUT geometry, which is hyper-Kähler. Therefore it admits a \mathbb{P}_1 of complex structures. The two different M5-brane components corresponding to the lifts of the D4/NS and the D4'/NS' systems correspond to M5-branes wrapped on two curves which are holomorphic in two different complex structures in this geometry. The rotation between the two complex structures in which the two curves are holomorphic is related to the amount of supersymmetry breaking (namely, to the angle between the D4- and D4'-branes in the type IIA configuration). Being holomorphic in some complex structure, each component is volume minimizing by itself. However, the complete system can be regarded as an M5-brane on a singular (i.e. reducible) non-holomorphic curve, which is therefore not volume-minimizing as a whole.

The M-theory state we build reduces at vanishing string coupling to the type IIA configuration we have studied. Interestingly, it captures various features of the IIA configuration such as its pseudo-moduli. Nevertheless, it is important to notice that this M-theory lift possesses non-holomorphic boundary conditions. The asymptotic behavior

⁷For general masses, the former system is still supersymmetric, while the latter breaks all supersymmetries unless the D6-branes for the N_c smallest masses are coincident.

of our M-theory curve differs from the one of the M-theory lift of the supersymmetric configuration (in particular it does not preserve supersymmetry even asymptotically). As a result, it cannot be interpreted as a state with spontaneously broken supersymmetry in a supersymmetric 4d theory. Instead, it corresponds to a state in a theory with a Lagrangian that breaks supersymmetry explicitly. This issue was investigated in detail in [19].

4.2 The curves

Following the argument above, we are led to describe the M-theory lift in terms of two curves which are holomorphic in two different complex structures of the Taub-NUT geometry (again, notice that this assumes a supersymmetric D4'/NS' system, hence that the corresponding N_c D6-branes are coincident).

In order to consider the lift of the D4/NS system, let us introduce an adapted complex structure, in which the corresponding M5-brane curve is holomorphic. In fact the system is locally $\mathcal{N} = 2$ supersymmetric, hence we may stick to the usual conventions for lifts of configurations of 4d $\mathcal{N} = 2$ theories [23]. Let us introduce $v = x_4 + ix_5$, $w = x_8 + ix_9$, and describe the ambient M-theory Taub-NUT geometry as the complex manifold

$$yz = \left[\prod_{i=N_c+1}^N (v - \mu_i) \right] (v - \mu)^{N_c} \tag{4.1}$$

where $(\mu_{N_c+1}, \dots, \mu_{N_f})$ correspond to the N largest mass parameters and $\mu < \mu_i$ is the common mass parameter for the N_c lightest flavors. Notice that the mass parameters encode the positions of the D6-branes (or of the Taub-NUT centers in M-theory) in the 4, 5 directions.

In these complex coordinates, the holomorphic curve corresponding to the D4/NS system has the structure

$$\begin{aligned} z - \prod_{i=N_c+1}^N (v - \mu_i) &= 0 \\ w &= 0 \end{aligned} \tag{4.2}$$

Hence, the M5-brane has spikes towards $z \rightarrow 0$ at the positions $v = \mu_i$. These spikes become the D4-branes upon reduction to type IIA. The interpretation for these spikes is that $v \rightarrow \mu_i$ corresponds to the cycle $yz = 0$ in the ambient Taub-NUT. This is reducible, and $z = 0$, y arbitrary, describe one component, corresponding to a spike ending on the Taub-NUT center (from the right).

The lift of the D4'/NS' component is also easily described, in the case of a common mass for the lightest flavors, on which we are centering. The system is locally $\mathcal{N} = 1$ supersymmetric, so it is described by a holomorphic curve in adapted complex coordinates. Intuitively we introduce $v' = x'_4 + ix'_5$, where x'_4 and x'_5 parametrize the 2-plane orthogonal to the x'_6 direction along which the D4'-branes stretch. Similarly we need to introduce new complex parameters μ'_i, μ' which encode the positions of the D6-branes in the v' direction. The mapping of complex coordinates in different complex structures (rotated

by the $SU(2)$ isometry of the Taub-NUT geometry, equivalently the $SO(3)$ rotation in the space parametrized by 4, 5, 6), and of different complex parameters specifying the D6-brane positions, is somewhat technical, but we provide its description in appendix A.

In the complex structure adapted to the $D4'/NS'$ system, the Taub-NUT geometry is described by

$$y'z' = \left[\prod_{i=N_c+1}^N (v' - \mu'_i) \right] (v' - \mu')^{N_c} \tag{4.3}$$

In these complex coordinates, the holomorphic curve describing the lift of the $D4'/NS'$ system is

$$\begin{aligned} z' - \prod_{i=1}^{N_c} (w' - w'_i) &= 0 \\ v' &= 0 \end{aligned} \tag{4.4}$$

Namely the M5-brane has spikes of order N_c towards $x'_6 \rightarrow -\infty$ (i.e. $z' \rightarrow 0$) at the position $w' = w'_i$. The parameters w'_i are free moduli of the holomorphic curve, and correspond to the mesonic field theory pseudo-moduli. Hence they remain as flat directions even in the M-theory lift.

It would be interesting to describe the lift of the type IIA configuration for completely general mass parameters, in particular when the $D4'/NS'$ system is non-supersymmetric by itself. We leave this interesting point for future work.

Being holomorphic in some complex structure, the above curves are automatically area-minimizing, and hence indeed correspond to the a classical stationary configuration for the M5-brane configuration, in the probe approximation. We will describe in section 5 the impact of possible backreaction effects in the curves, and on the pseudo-moduli stabilization.

We conclude with a final remark. Notice that the positions of the D6-branes in 6 enter in the determination of the complex parameters μ' , μ'_i , and hence appear in the expression of the curve. It is a familiar fact that in supersymmetric vacua such positions are hidden parameters of the brane configuration which are not visible in holomorphic quantities of the gauge theory. Hence it is not unexpected that they pop up as relevant quantities when dealing with a non-supersymmetric vacuum. It would be interesting to gain a better understanding of the interplay of gauge theory quantities and these parameters.

4.3 A heuristic argument

We would like to conclude with a suggestive heuristic derivation of the factorized structure. In the field theory description, the ISS construction of the local minimum can be obtained by starting with the electric theory, moving on to the magnetic dual and taking the limit in which the gauge interactions vanish. This procedure can be carried out in the M-theory description of the configuration, providing independent evidence for the factorized structure.

The infrared physics of the electric theory is described by the M5-brane wrapped on the holomorphic curve obtained in [14],

$$t = w^{N_c - N_f} (w - w_0)^{N_f} \tag{4.5}$$

$$v w = m w_0 \tag{4.6}$$

with

$$w_0 = \left(\frac{\Lambda_{\text{SQCD}}^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c} \tag{4.7}$$

where t is related to the usual variable used in $\mathcal{N} = 2$ configurations by a rescaling by μ^{N_c} , with μ the adjoint mass that goes to infinity in the $\mathcal{N} = 1$ limit. The value of w_0 determines the expectation value of the mesons at the supersymmetric vacua. It is straightforward to rewrite w_0 in terms of magnetic quantities.

The two equations defining the curve can be rewritten to clarify how the curve relates to the lifts of the NS and NS'. Rewriting (4.5) as

$$t - w^{N_c - N_f} (w - w_0)^{N_f} = 0 \tag{4.8}$$

we see N_c D4-branes ending on the NS'-brane (from the left). Combining (4.5) and (4.6) we obtain

$$\left(\frac{m^{N_f - N_c}}{w_0^{N_c}} v^{N_c} \right) t - (m - v)^{N_f} = 0 \tag{4.9}$$

showing N_c (resp. N_f) D4-branes ending on the NS-brane (from the right resp. left).

As mentioned above, in the field theory one constructs the non-supersymmetric vacuum by going to the magnetic dual and turning off the magnetic gauge interactions. In order to heuristically perform this operations in terms of the M5-brane curve, it is useful to rewrite quantities in terms of magnetic variables, which are related to the electric ones by

$$\Lambda_{\text{SQCD}}^{3N_c - N_f} \Lambda^{3(N_f - N_c) - N_f} = \hat{\Lambda}^{N_f} \quad ; \quad h = \Lambda / \hat{\Lambda} \quad ; \quad \mu^2 = -m \hat{\Lambda} \tag{4.10}$$

We then have

$$m = -\frac{h\mu^2}{\Lambda} \quad ; \quad w_0 = \left(\frac{\Lambda^{N_c + (3N_c - 2N_f)}}{(-\mu^2)^{N_c - N_f} h^{N_c}} \right)^{1/N_c} \tag{4.11}$$

$$m w_0 = ((-\mu^2)^{N_f} \Lambda^{3N_c - 2N_f})^{1/N_c} \tag{4.12}$$

The non-supersymmetric vacuum appears in the magnetic theory when we take the classical limit, turning off the gauge interactions. Hence, we are interested in the limit $\Lambda \rightarrow 0$ with h and μ fixed. In this limit

$$\begin{aligned} m &\sim \Lambda^{-1} && \rightarrow \infty \\ w_0 &\sim \Lambda^{1 + (3N_c - 2N_f)/N_c} && \rightarrow 0 \\ m w_0 &\sim \Lambda^{(3N_c - 2N_f)/N_c} && \rightarrow 0 \end{aligned} \tag{4.13}$$

where we have explicitly used that we are working on the free magnetic range $N_c + 1 \leq N_f < \frac{3}{2}N_c$. With this, (4.6) becomes

$$v w \rightarrow 0 \tag{4.14}$$

We thus see that the curve splits into two components. One of them (associated with the D4'/NS' system in the type IIA configuration) corresponds to $v = 0$ and is given by the limit of (4.8). The other one (associated with the D4/NS system) has $w = 0$ and is given by the limit of (4.9). These curves have the expected behavior i.e. for the first one $t \sim w^{N_c}$ as $w \rightarrow \infty$ and for the second one $t \sim v^{N_f - N_c} = v^{N_f}$ as $v \rightarrow \infty$.

The argument is heuristic since the curve cannot completely agree with the complete M5-brane curve that we have determined in previous sections. This is because the holomorphic curve remains holomorphic in the limit. However, the naive translation of the ISS field theory construction to the M5-brane curve does have a suggestive structure. Indeed the curve reproduces the correct structure to the best extent that one can expect from a holomorphic curve! Namely, it factorizes into two components which have the correct number of spikes ending on the correct NS/NS' fivebranes. The only caveat is that the two components are holomorphic in the same complex structure. Our interpretation is that, since the holomorphic curve of the supersymmetric vacuum is insensitive to the positions of the Taub-NUT centers in the x_6 direction, it reproduces the correct structure for the non-susy vacuum in the limit where the centers are sent off to $x_6 \rightarrow -\infty$ (in which it becomes holomorphic). In a sense, it is the only regime where the holomorphic curve can be expected to match the M5-brane curve of the non-supersymmetric vacuum.

In order to have a closer look at the factorized holomorphic curve arising in the limit, it is useful to introduce the rescaled variables

$$\begin{aligned} \tilde{t} &= t/w_0^{N_c} \\ \tilde{v} &= v/m \\ \tilde{w} &= w/w_0 \end{aligned} \tag{4.15}$$

The two components that follow from (4.8) and (4.9) become

$$\tilde{t} - \tilde{w}^{N_c} = 0 \tag{4.16}$$

$$\tilde{v}^{N_c} \tilde{t} - (1 - \tilde{v})^{N_f} = 0 \tag{4.17}$$

They represent the two components elongating to infinity with different asymptotic behavior in \tilde{v} . It is easy to realize that they agree with the two components of our M5-brane curve in previous sections, in a suitable limit (in which in particular the two complex structures become the same).

5. Pseudo-moduli stabilization

We have shown that the field theory pseudo-moduli correspond to geometric moduli of type IIA configuration. They moreover remain flat directions in the M-theory configuration, at least in the case where the D4'/NS' system is supersymmetric, where we could determine

the structure of the curve. Hence the quantum gauge theory effects encoded in the M-theory curves do not include the 1-loop correction lifting these accidental flat directions.

In fact it is easy to understand what the mechanism responsible for the stabilization is. The fields that contribute to the field theory 1-loop Coleman-Weinberg potential are classically massive fields whose mass depends on the pseudo-moduli. In the type IIA brane configuration, the pseudo-moduli are the geometric positions of D4'-branes in 8,9. Clearly, the classically massive fields whose mass depends on these positions are D4-D4' and D4'-D4' open strings. However, the D4'-D4' open strings are not sensitive to the breaking of supersymmetry and do not contribute, hence only the D4-D4' states contribute. The Coleman-Weinberg potential then corresponds to the annulus diagram with boundaries on the D4- and D4'-branes. Hence the lifting of pseudo-moduli is an effect that cannot be detected from the study of the D4/NS or D4'/NS' systems in isolation, but which arises from their interaction.

Due to the complicated geometry (and the presence of the NS- and NS'-branes) this diagram cannot be computed for arbitrary locations of the D4'-branes. In the small distance regime, its result should reproduce the field theory result. Unfortunately the brane configuration does not seem to provide new insights in this regime. On the other hand, in the large distance regime, the annulus diagram corresponds to the exchange of supergravity modes (graviton, dilaton and 5-form exchange) between the D4- and D4'-branes. Being non-supersymmetric, it is expected that the gravitational exchange overcomes the RR-form repulsion (which is smaller due to the misalignment of the D-branes) and lead to a net attraction, which pushes the D4'-branes towards the origin in 8,9. This is the string theory view of the lifting of the pseudo-moduli, in the large field region of pseudo-moduli space.

The above discussion can be lifted to M-theory. Each M5-brane component is area-minimizing by itself, and the component corresponding to the lift of the D4'/NS' system has arbitrary moduli. Their lifting can be described in terms of the interaction between the two components, which in the long distance regime reduces to graviton and 3-form exchange. This implies that the lifting of moduli requires describing the configuration beyond the brane probe approximation. In principle, a quantitative computation of this effect could be achieved by considering the backreaction of the M5-brane component associated with the D4/NS system, and solving the area minimization equations, in the backreacted background, for the M5-brane component associated with the D4'/NS' system. A similar approach was used in [14], where the above procedure to compute supergravity interactions between different M5-brane components provided certain correction to the metric on the Higgs branch of $\mathcal{N} = 2$ gauge theories. The reduced (in fact, absence of) supersymmetry in our present problem clearly suggest that the computation is might be considerably more difficult and beyond the scope of this paper. We hope to come back to this point in future work.

6. Symplectic and orthogonal gauge groups

It is straightforward to carry out a similar discussion for the non-supersymmetric minima

in the $SO(N_c)$ and $USp(N_c)$ theories with N_f massive flavors. In fact, the type IIA configurations realizing these gauge theories, and their Seiberg duality properties, have been studied in [24]. We sketch the new features of this construction, referring the reader to this reference (see also [13] for a review) for details.

The construction of the electric theories combines the same ingredients as for the $SU(N_c)$ theory (namely, N_c D4-branes suspended between an NS- and an NS'-brane, in the presence of N_f D6-branes), plus an additional O4-plane, stretching along the directions 01236 (i.e. parallel to the D4-branes). The O4-plane flips its charge as it crosses the NS- and NS'-branes.

The introduction of the O4-plane pairs up the D4-branes in two sets, related by the orientifold symmetry, and reduces their gauge symmetry down to $SO(N_c)$ or $USp(N_c)$ when the middle piece of the O4-plane has negative or positive charge, respectively. Similarly, the D6-branes pair up and reproduce the appropriate global symmetries for these gauge theories. Finally notice that for odd N_c the $SO(N_c)$ configuration has an unpaired D4-brane on top of the middle part of the O4-plane, while for the $USp(N_c)$ configuration this is not possible (see [25, 26] for details). Notice that our notation for the number of D-branes is dictated by counting on the covering space. In this convention, the O4-plane charge is ± 4 D4-brane charge units.

Seiberg duality is obtained by moving the NS across the D6- and the NS'-brane. In the process, there is a change in the number of D4-branes which determines the final number N of D4-branes joining the D6-branes and the NS-brane (which controls the rank of the Seiberg dual gauge group). Since there is a contribution of the O4-plane charge to this Hanany-Witten effect, one obtains $N = N_f - N_c + 4$ for SO and $N = N_f - N_c - 4$ for USp .

Using these rules one can directly construct the type IIA configuration corresponding to the non-supersymmetric minima for the SO and USp theories described in [1]. It is shown in figure 6. As in the SU case, it is possible to match all classical properties of the field theory with geometric properties of this configuration.

An important difference is that the D4'/NS' system is non-supersymmetric by itself (even in the case of equal flavor masses). In particular one may be worried by the fact that we have several D4-branes at angles which seemingly intersect (as they reach the NS'-brane) and could potentially lead to tachyons. One may think that the presence of the O4-plane imposes an orientifold projection that removes them; however, given that the tachyonic modes have a matrix structure, they cannot be completely removed by such orientifold projection, which at most projects the matrix down to the symmetric or antisymmetric components. The key ingredient must therefore be the presence of the NS'-brane. Indeed, the presence of such object at the coincidence of the D4-branes can prevent the naive claim that there is an open string tachyon in the D4-D4 spectrum. In the following we assume that this is indeed the case (as suggested by its agreement with the field theory picture that no instability exists) and proceed to use this additional rule in our subsequent examples. It would be interesting to provide further support for it based on computations of open string spectra in the near horizon region of NS-branes as in [27].

As in the SU case, the construction shows that the lift to M-theory is given by an M5-brane wrapped on a reducible curve. An important difference is that, since the D4'/NS'

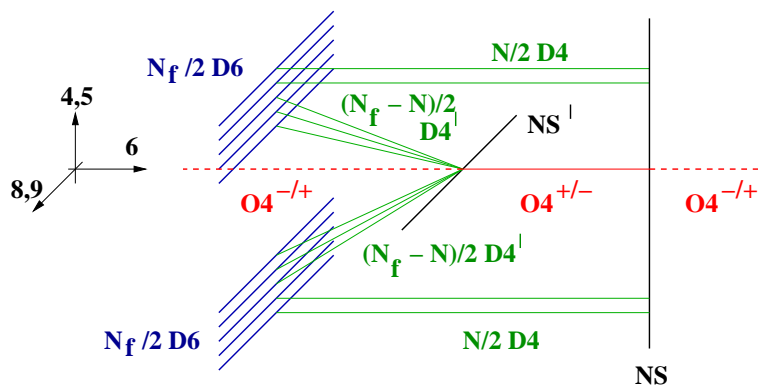


Figure 6: The type IIA brane configuration for the non-supersymmetric minimum of the $SO(N_c)$ and $USp(N_c)$ gauge theories with N_f massive flavors. For clarity we have shown the situation for different flavor masses. The red line corresponds to the O4-plane, and its change of charge as it crosses the NS- and NS'-branes is shown as a change between dashed and solid. For the $SO(N_c)$ theory we have $N = N_f - N_c + 4$, and it corresponds to choosing the O4-plane to have negative charge in the middle interval, and positive on the semi-infinite pieces. For the $USp(N_c)$ theory we have $N = N_f - N_c - 4$ and the O4-plane charge is positive in the middle interval.

system is non-supersymmetric by itself (even in the case of equal flavor masses), it lifts to an M5-brane component which is not holomorphic (in any complex structure). We leave this discussion for future work.

7. Generalizations

The realization of known non-supersymmetric local meta-stable minima in terms of brane configurations leads to a precise identification of the key ingredients in this phenomenon. In this section we use this ingredients to construct non-supersymmetric local meta-stable minima in other field theories which admit a realization in terms of type IIA brane configurations.⁸ Clearly there are many other possibilities, which we leave as an exercise for the reader.

$SU(N_c)$ with non-chiral matter in the \square or $\bar{\square}$

The type IIA brane realization of the $SU(N_c)$ with non-chiral matter in symmetric or antisymmetric (plus additional flavors) has been achieved in [28], by the introduction of O6-planes (along 0123789). Using the ingredients in these configurations, and the basic building blocks of non-supersymmetric meta-stable minima, it is straightforward to construct a brane configuration realizing a non-supersymmetric meta-stable vacuum in these theories. The configuration is shown in figure 7.

⁸We do not study the supersymmetric vacua, longevity of the meta-stable minimum, etc. This would be an interesting exercise. We consider the similarity of the brane configurations with those in the previous sections makes it clear that these issues will not change much.

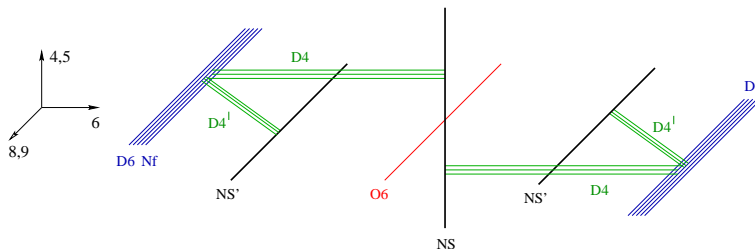


Figure 7: The type IIA brane configuration describing the non-supersymmetric meta-stable minimum in the $SU(N_c)$ theory with non-chiral matter in the symmetric or antisymmetric representations (corresponding to the choice of positive or negative O6-plane charge), plus massive flavors.

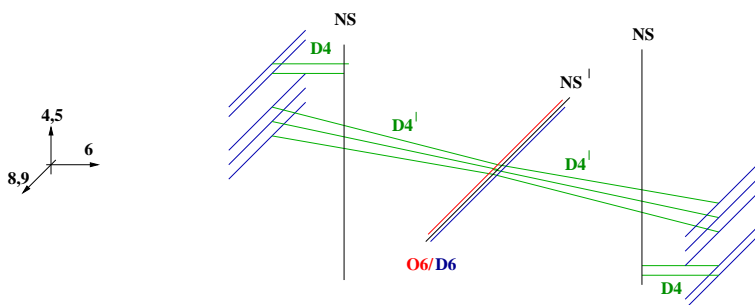


Figure 8: The type IIA brane configuration describing the non-supersymmetric meta-stable minimum in the chiral $SU(N_c)$ theory with matter in antisymmetric, conjugate symmetric and fundamental representations. Here O6/D6 stands for the system of the split O6-plane and the 8 half D6-branes.

Chiral $SU(N_c)$ theory with chiral multiplets in the $\square + \overline{\square} + 8\square$

We would like to present one example of a chiral theory with a non-supersymmetric meta-stable vacuum. Although type IIA brane configurations are not particularly well suited for the construction of chiral theories (and configurations of D3-branes at singularities may provide a better starting point [3]), there is a type IIA configuration realizing a chiral $SU(N_c)$ theory with one chiral multiplet in the antisymmetric, one in the conjugate symmetric, and 8 in the fundamental representations [29–31]. The configuration contains two NS- and one NS'-brane, with an O6-plane passing through the latter. The key ingredient for producing chirality is that the O6-plane is split into two pieces by the NS'-brane, with both pieces carrying different charge. In order to cancel an NS'-brane worldvolume tadpole, one needs to introduce 8 half D6-branes ending on the latter (see related discussions in [32, 33]).

The construction of the configuration realizing the non-supersymmetric meta-stable vacuum is fairly easy. The only subtlety is that, since the NS'-brane must be on top of the O6/D6 system, it is convenient to use the configuration where the local minimum is described in the electric theory. The configuration is shown in figure 8, where O6/D6 stands for the system of the split O6-plane and the 8 half D6-branes.

As should be clear by now, any type IIA brane configuration can be modified to include the basic ingredients involved in the appearance of the non-supersymmetric meta-stable minima. It is therefore straightforward to generalize to product gauge group theories, etc.

We refrain from entering a more detailed discussion of these possible generalization, leaving them for the interested reader.

A last word of caution is appropriate. The meta-stable vacua we are studying occur at small expectation values of fields. In order to be certain of their existence it is thus crucial to have the Kähler potential under control such that we can ensure that it remains regular. This is achieved in these examples when the macroscopic magnetic description of the theory is IR free. This is easily attainable in the $SO(N)$ and $USp(N)$ examples of section 6. From the point of view of the electric theory, the free magnetic ranges correspond to $N_f < \frac{3}{2}(N_c - 2)$ for $SO(N_c)$ with N_f flavors and $N_f < \frac{3}{2}(N_c + 1)$ for $USp(N_c)$ with $2N_f$ flavors. We are confident that it is possible to find free magnetic ranges in generalizations such as the theories described in this section by appropriately tuning the numbers of colors and flavors in the magnetic theories. Most of what is currently known about many of these field theories comes from their realization by means of D-brane setups, which yield no information about the free magnetic and other ranges. On the other hand, our experience with the factorization of the M-theory curve in section 4.3 suggests that some understanding of these problems might follow from the M-theory lift.

8. Final remarks

In this paper we have described the construction of type IIA brane configurations which realize non-supersymmetric meta-stable vacua of 4d $\mathcal{N} = 1$ supersymmetric gauge theory. The realization of these vacua for the field theory examples in [1] allows us to identify the key ingredients of the brane configuration related to the existence of these vacua. And hence to generalize the construction to many other brane configurations and field theories.

We have also provided a description of the lift of these configurations to M-theory, in terms of M5-branes wrapped on a reducible curve. In the simplest situation (where some mass parameters are equal) its components are holomorphic in different complex structures of the underlying Taub-NUT geometry. We have argued that a complete understanding of the physics of the local minimum, in particular of the lifting of the pseudo-moduli, requires a description beyond the brane probe approximation, namely taking into account the interaction between the two components. The quantitative treatment of this problem thus remains an important open issue in these constructions.

It would be interesting to find connections between the constructions we have presented in this paper, and the discussion of non-supersymmetric meta-stable vacua for gauge theories on systems of D-branes at singularities. It is possible that T-duality relations along the lines of [34, 35] provide a bridge between both languages.

We expect much progress in the understanding of these non-supersymmetric meta-stable vacua from their realization in string theory.

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A. The hyper-Kähler structure of Taub-NUT and rotation of the holomorphic structure

A.1 Hyper-Kähler construction of the multicenter Taub-NUT spaces

Let us describe the (multicenter) Taub-NUT space as a hyper-Kähler quotient, following the reasoning in [36]. In order to build the Taub-NUT in this way, we start from the manifold \mathcal{M} given by $d + 1$ copies of \mathbf{H} , where d is the number of centers in our space and \mathbf{H} is a copy of \mathbf{R}^4 with flat hyper-Kähler metric. Let us take as coordinates in \mathcal{M} the quaternions w and q_a , where a goes from 1 to d .

Now consider the abelian group G of rank d acting on the manifold, this group is isomorphic to \mathbf{R}^d locally. The moment map for this group acting on \mathcal{M} is given by

$$\mu_a = \frac{1}{2} \mathbf{r}_a + \mathbf{y}, \tag{A.1}$$

where $\mathbf{r}_a = q_a i \bar{q}_a$ (no sum in a , and boldface denotes three dimensional vectors) and $\mathbf{y} = (w - \bar{w})/2$. Under the a -th factor of G , q_a transforms with a $+1$ U(1) charge, w gets translated and the rest of the coordinates remain invariant. Let us consider the set of vectors \mathbf{e}_a . Then we define our Taub-Nut space as:

$$X = \mu^{-1}(\mathbf{e})/G, \tag{A.2}$$

where the a index is implicit. Namely, we consider all points in \mathcal{M} such that their moment maps μ_a give \mathbf{e}_a , and then quotient the resulting space by the action of G . With the metric inherited from the flat \mathcal{M} one gets the multi center Taub-NUT space with the standard metric:

$$ds^2 = \frac{1}{4} V d\mathbf{r}^2 + \frac{1}{4} V^{-1} (d\tau + \vec{\omega} \cdot d\mathbf{r})^2, \tag{A.3}$$

with $\nabla \times \vec{\omega} = \vec{\nabla} V$ and

$$V = 1 + \sum_{a=1}^d \frac{1}{|\mathbf{r} - \mathbf{e}_a|}, \tag{A.4}$$

so we can identify the values of the moment maps with the positions of the centers of the Taub-NUTs.

A.2 Complex structure for these spaces

In this section we will try to understand better the structure as a complex manifold of the space we just built following [23]. Recall that a quaternion can be written as $q = a + ib + jc + kd$, where i, j and k satisfy the SU(2) Lie group algebra, so we can think of them as the Pauli matrices, and a, b, c and d are real numbers. This structure reflects the hyper-Kähler nature of the manifold, we can associate choosing a complex structure (in the S^2 of possible complex structures) with privileging i , say, and then decomposing the quaternion q into two complex numbers w_1 and w_2 given by:

$$w_1 = a + ib \tag{A.5}$$

$$w_2 = c + id, \tag{A.6}$$

the decomposition being motivated by $q = a + ib + j(c + id)$. We have the freedom of choosing any combination of i , j and k as determining a complex structure, so we have essentially the freedom of choosing a direction in \mathbf{R}^3 . Note also that the 3-vector structure of the moment maps also comes from ijk , so rotating the directions of the base space for the Taub-NUT (i.e., ordinary rotations in the type IIA picture) roughly corresponds to choosing different complex structures. This will be the basic idea in what follows.

Let us privilege a complex structure and then separate q_a into the complex variables y_a and z_a , and w into v and v' , such that the action of G in these complex variables is given by:

$$y_a \rightarrow e^{i\theta_a} y_a \tag{A.7}$$

$$z_a \rightarrow e^{-i\theta_a} z_a \tag{A.8}$$

$$v \rightarrow v \tag{A.9}$$

$$v' \rightarrow v' - \sum_{a=1}^d \theta_a. \tag{A.10}$$

Also, when we pick a complex structure, namely a direction in 3 space, the moment map can be divided into the longitudinal part (a real part $\mu_{\mathbf{R}}$) and a transverse (complex) part $\mu_{\mathbf{C}}$. In the type IIA picture the latter corresponds to the projection of the D6 brane position into the 4,5 plane in which the NS brane is sitting, and the former to the x^6 position of the brane, which as we will see below does not appear in the defining equations for the NS factor of the M theory curve in the complex structure in which it is holomorphic.

The components of $\mu_{\mathbf{C}}$, a give the equations:

$$y_a z_a = v - e_a, \tag{A.11}$$

where e_a is the projection of \mathbf{e}_a in the 4,5 plane. We can define then the manifold X in terms of the G invariants and any constraints between them. In terms of the invariants $y = e^{iv'} \prod_{a=1}^d y_a$, $z = e^{-iv'} \prod_{a=1}^d z_a$ and v , the defining equation for the resulting space is given by:

$$yz = \prod_{a=1}^d (v - e_a), \tag{A.12}$$

which is the equation we have been using in the main text for the Taub-NUT.

A.3 Rotating the complex structure

We have described how to obtain the equations for the Taub-NUT in a given complex structure, but in our system there are two different relevant complex structures with no holomorphic relation between them, and we expect that the y , z and v parameters describing the Taub-NUT in the complex structure in which the NS factor of the M theory curve is holomorphic have a complicated non-holomorphic relation with the parameters y' , z' and v' describing the curve in the complex structure where the NS' factor is holomorphic. In this section we describe how to obtain explicit relations between both sets of coordinates.

The basic idea has already been described. What we notice is that rotations of the moment maps have two interpretations, one as rotations in the space of complex structures of the hyper-Kähler manifold and the other as rotations in the Type IIA theory. Since in the type IIA theory the rotation necessary for going from the direction associated with the NS factor being holomorphic (i.e., x^6) to the direction in which the NS' factor is holomorphic ($x^6 \cos \theta + x^4 \sin \theta$, where θ is the rotation angle, given by the masses and the position in x^6 of the D6 branes) is easy to determine with simple trigonometry, the appropriate change in complex structure is simple to determine too. For example, let us identify the ijk directions in quaternion space with the 6, 4, 5 directions in the type IIA picture. With this convention, the complex structure making the NS factor holomorphic is given by privileging i , and splitting the quaternionic coordinates as $q_a = (a + ib) + j(c + id) = (y_a, z_a)$. Now we rotate in order to obtain the expressions in the coordinates where NS' is holomorphic. The effect in ijk is given by:

$$\begin{pmatrix} i \\ j \\ k \end{pmatrix} \rightarrow \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} i \\ j \\ k \end{pmatrix} \equiv \begin{pmatrix} I \\ J \\ K \end{pmatrix}. \tag{A.13}$$

We now want to split the moment maps into complex coordinates where I is the privileged complex structure. We just substitute and read components, let us do it for some generic q :

$$q = a + bi + cj + dk = a + b(I \cos \theta + J \sin \theta) + c(J \cos \theta - I \sin \theta) + dk \tag{A.14}$$

$$= a + I(b \cos \theta - c \sin \theta) + J(b \sin \theta + c \cos \theta + Id). \tag{A.15}$$

From here we read what the new y and z are, and since we have the expressions of $abcd$ in terms of the original y and z (for example, $b = -i/2(y - \bar{y})$), this completely determines the new variables as non-holomorphic functions of the old ones and the θ angle.

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Erratum

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